Diagnostics: unusual observations & partial plots

ST552 Lecture 17

Charlotte Wickham

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Case Influence Statistics

→ observation

A selection of metrics for observations that help identify *unusual observations*. → doesn't mean an assumption is violated → model level

Three kinds of unusual observations:

- **High leverage observations** are unusual in their combination of explanatory values and have the potential to be influential.
- **Outliers** don't fit the model well (their combination of response and explanatories is unusual according to the model)
- **Influential observations** substantially change the model when included/excluded. We don't want out conclusions to rely heavily on a few influential observations. Generally are also one of high leverage and/or outliers.

BEWARE!

Case Influence Statistics

There are many metrics. We'll cover one for each kind of unusual observation but be aware there are others.

Don't worry about the formulas, concentrate on the concept.

Faraway 6.2

NOT

EXAMINABLE

Leverage

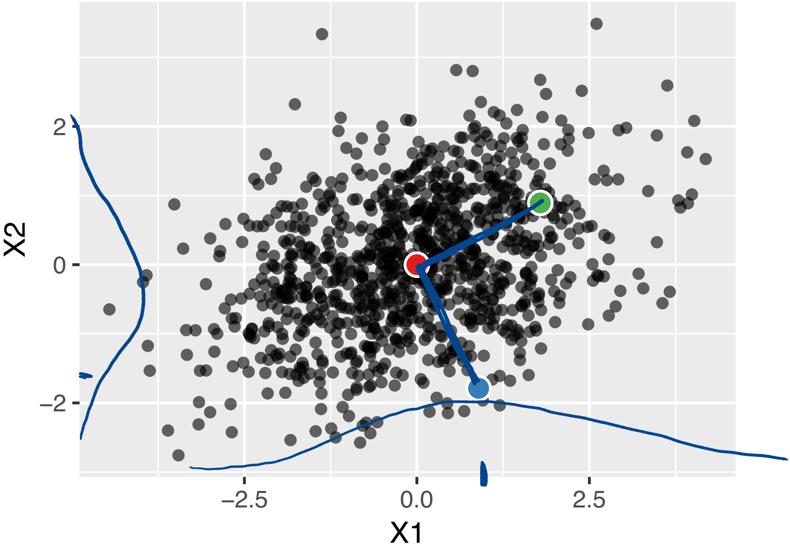
The leverage of a observation is $h_i = \boxed{H_{ii}}$ the ith diagonal element of the hat matrix, H. $= X(X^TX)^{-1}X^T$ $\hat{y} = Hy$

Remember the hat matrix only depends on X.

h_i is a Mahalanobis distance (wikipedia it).

It's a measure of how far the observations explanatory values are from the mean of the observed explanatory values, but it takes into account the unequal variances of each explanatory variable and their correlations.

The regression line/surface is pulled towards observations with high leverage.



Blue point is much further from the mean in the Mahalanobis sense than the green point. The blue point would have higher leverage.

Outliers

A observation that doesn't fit the regression model will have a large error, but this doesn't necessary translate to a large residual because we fit the model to minimize residuals.

Instead of just looking for large residuals, compare the predicted response from a model without the observation (so it can't influence the model), to the observed response,

$$\boxed{\hat{y}_{(i)} - y_i}$$

observed response

where $\hat{y}_{(i)}$ is the fitted value for the ith ~~observations~~ observation from a model fitted to the data excluding the ith observation.

This value, appropriately standardized, is called the **Studentized residual**.

Influence

How much does the model fit change when the observation is excluded? A substantial change indicates an influential observation. The distance between the vector of fitted values when the ith observation is excluded and fitted values when the ith observation is included is:

$$(\hat{\mathbf{y}} - \underbrace{\hat{\mathbf{y}}_{(i)}}_{\text{vector}})^T(\hat{\mathbf{y}} - \hat{\mathbf{y}}_{(i)}) \qquad \sum_{j=1}^{n} (\hat{y}_j - \hat{y}_{(i),j})^2$$

This value, appropriately scaled, is called **Cook's Distance**.

" change in fitted values when you exclude the observation from the model "

Handout

In each plot:

1. Draw in a fitted regression line including the point marked with $+$, and a fitted regression line excluding the point marked with $+$
2. Decide if the point marked with $+$ would be high leverage, an outlier, and/or influential.

Case influence: examples

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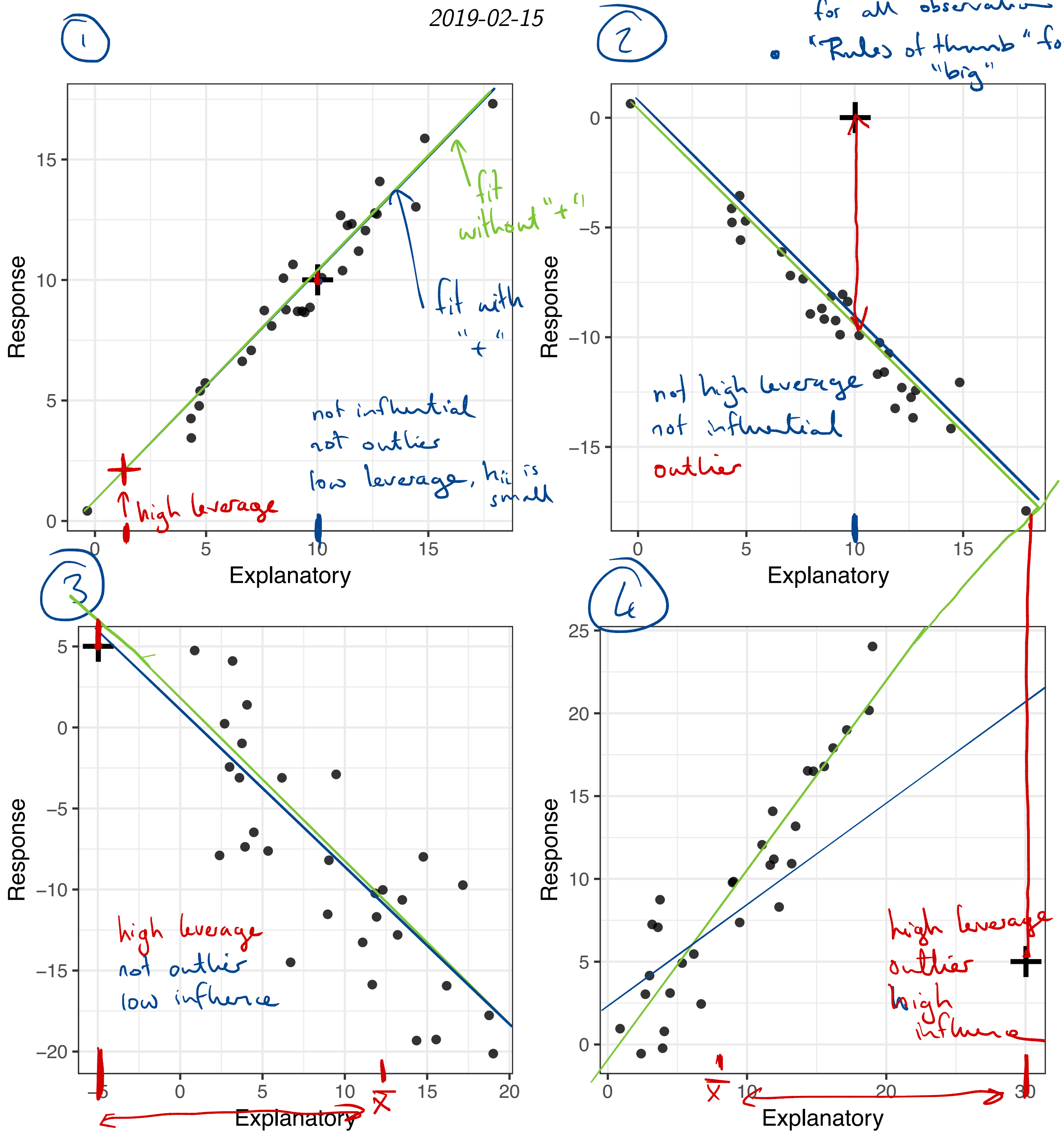
Upcoming homework:
- Plot these measures for all observations
- "Rules of thumb" for "big"



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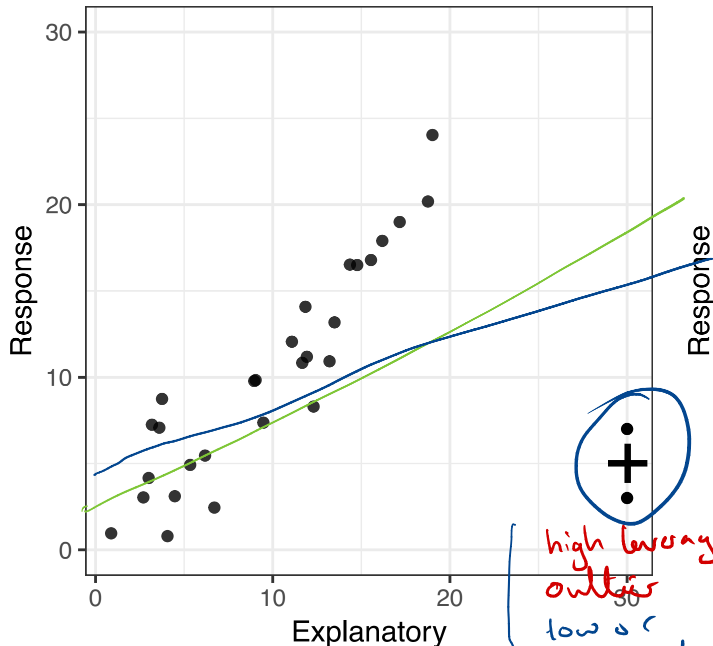
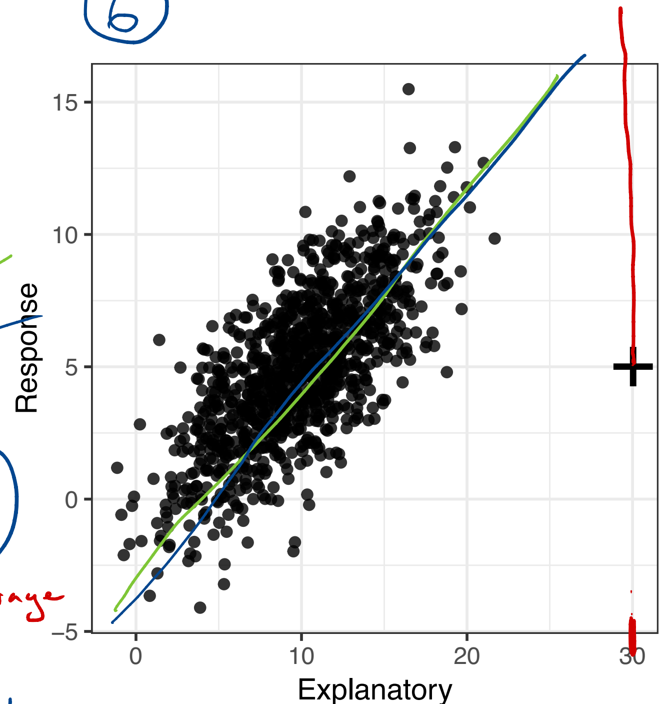


high leverage
outlier
low or
moderate
influence

don't pick "groups" of
unusual points

6



A single high leverage
outlying point isn't
always influential

Faraway 6.3

Assumptions: $E[Y] = X\beta$

Motivation: We can examine a plot of the response against each explanatory, but the observed relationship is complicated by the effect of the other variables.

residuals vs. ~~fitted values~~ explanatories

Useful to graphically check relationships between the response and explanatories after "accounting for" the other variables. Examine for evidence of non-linearity and unusual observations.

Partial regression plots For explanatory variable, j:
Regress y on all explanatories except the jth and find residuals, $\hat{\delta}$
Regress x_j on all explanatories except itself and find residuals, $\hat{\gamma}$
Plot $\hat{\delta}$ against $\hat{\gamma}$

Both cases: the SLR its slope will match estimated coefficient

Partial residual plots

For explanatory variable, j: Plot

$X\hat{\beta}$ but without $x_j \hat{\beta}_{j-1}$

$$y_i - \sum_{i \neq j} x_{ij} \hat{\beta}_j$$

obs response

against the jth explanatory variable.