Joint confidence intervals

ST552 Lecture 10

Charlotte Wickham

January 27, 2016

Today

- Confidence intervals for parameters jointly
- Introduction to prediction

Review: Your Turn

```
##                Estimate Std. Error    t value    Pr(>|t|)
## (Intercept) -28.8767696  19.735418 -1.4631952 0.155399149
## Acetic        0.3277413   4.459757  0.0734886 0.941979774
## H2S           3.9118411   1.248430  3.1334077 0.004247081
## Lactic       19.6705434   8.629055  2.2795710 0.031079481
```

Construct a 95% confidence interval for β_2 (corresponds to the H2S term) in the cheddar example.

(Hint: `qt(0.975, 26)` $= 2.06$)

Write a one sentence interpretation.

Joint confidence regions

A joint $100(1-\alpha)\%$ confidence for the vector β can be formed using,

$$(\hat{\beta}-\beta)^T X^T X(\hat{\beta}-\beta) \leq p\hat{\sigma}^2 F_{p,n-p}^{(\alpha)}$$

and results in p-dimensional ellipsoids (very hard to visualise, but essential for communicating joint uncertainty when the parameters are correlated).

The 2D ellipsoid example

See `09-cheddar.R`

Takeaways:

- Joint confidence regions are ellipsoid.
- Essential for communicating the correlation between the estimates.
- There is a duality between the joint confidence region and the F-test. If the null hypothesis on the parameters lies inside the 95% confidence region, the F-test would fail to reject the null at the 5% level.

Relationship between slope and intercept

See `09-cheddar.R`

Takeaways:

- The intercept estimate is generally correlated with the other parameter estimates.
- Centering the covariates doesn't change the model (the parameter estimates change, but their meaning is different, fitted values are identical.)
- Centering removes the correlation between the intercept estimate and the other parameter estimates.

Prediction

We've built a model:

$$y = X\beta + \epsilon$$

Now given a new vector of values of the explanatories x_0 we can predict the response

$$\hat{y_0} = x_0^T \hat{\beta}$$

But what is the uncertainty in this prediction?

Two kinds:

- prediction of the mean response
- prediction of a future observation

Friday

I'm out of town.

Matt will cover lecture.

Bring a laptop if you can.