Inference in regression: confidence intervals

ST552 Lecture 9

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Today

- Finish last time's slides and exercises from last time
- Another view of F-tests
- Confidence intervals for single parameters
- Confidence intervals for linear combinations of parameters
- Confidence intervals for parameters jointly

Last time...

Certain hypotheses of interest can be set up as competing models. A full model and a simpler model (nested in the full model). A.K.A testing models.

Identify the models of interest. Fit both. Check fit of full model. Find F-statistic, and answer questions of interest.

Another way to set up F-tests A.K.A testing linear parametric functions

Assuming the regression model:

$$Y = X\beta + \epsilon, \quad \epsilon \sim N(0, \sigma^2 I)$$

Consider the hypotheses:

you specify

$$H_0 : K^T\beta = m$$
$$H_1 : \underset{k\times p}{K^T}\,\underset{p\times 1}{\beta} \neq \underset{k\times 1}{m}$$

where $K^{T}_{k\times p}$ matrix with rank(K) = k.

Then under the null hypothesis,

$$F = \frac{\left((K^T\beta - m)^T \left(K^T(X^TX)^{-1}K\right)^{-1}(K^T\beta - m)\right)/k}{\text{RSS}/(n-p)} \sim F_{k,n-p}$$

This alternative is equivalent to the model testing set up we considered. Every null hypothesis of the form $K^T\beta = m$ is comparing a full and reduced model and vice versa.

For example, consider

$$K = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{p \times 1}, \quad m = \mathbf{0}$$

j'th position counting from zero

$k = 1$

$K^T \underset{\sim}{\beta} = \beta_j = 0$

Single parameter test

Your turn

What are K and m for exercises 1 and 5 from the handout from last time?

Confidence intervals for individual β_j and linear combinations

The t-test for an individual parameter can be flipped around to give $100(1-\alpha)\%$ confidence intervals of the form

$$\hat{\beta}_j \pm t_{n-p}^{(\alpha/2)} \text{SE}\left(\hat{\beta}_j\right)$$

(Remember $\text{SE}\left(\hat{\beta}_j\right)$ is coming from the diagonal entry of the estimated variance-covariance matrix.)

Similarly, confidence intervals for a linear combination of the parameters, $c^T \beta$ where $c_{p\times 1}$, can be formed

$$c^T \hat{\beta} \pm t_{n-p}^{(\alpha/2)} \sqrt{\hat{\sigma}^2 c^T (X^T X)^{-1} c}$$

$$\underset{p\times 1}{c} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad c^T \beta = \beta_j$$

$$\beta_1 - \beta_2 \qquad c = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix}$$

$$\beta_0 + \beta_1 5 + \beta_2 10 + \beta_3 15 + \dots + \beta_p 20$$

also works for prediction

Joint confidence regions

$$\leq t_{n-p}^{(\alpha/2)}$$

$$\frac{\hat{\beta}_j - \beta_j}{SE(\hat{\beta}_j)} \sim t_{n-p}$$

A joint $100(1-\alpha)\%$ confidence for the vector β can be formed using,

$$(\hat{\underset{\sim}{\beta}} - \underset{\sim}{\beta})^T X^T X (\hat{\underset{\sim}{\beta}} - \underset{\sim}{\beta}) \leq p\hat{\sigma}^2 F_{p,n-p}^{(\alpha)}$$

and results in p-dimensional ellipsoids (very hard to visualise, but essential for communicating joint uncertainty when the parameters are correlated).