

Inference in regression: F-test

ST552 Lecture 8

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Today

- ▶ The F-test
- ▶ Practice with F-tests

F-test

Let Ω denote a larger model of interest with p parameters and ω a smaller model that represents some simplification of Ω with q parameters.

Intuition: If both models “fit” as well as each other, we should prefer the simpler model, ω . If Ω shows substantially better fit than ω , that suggests the simplification is not justified.

How do we measure fit? What is substantially better fit?

F-statistic

$$F = \frac{(\text{RSS}_\omega - \text{RSS}_\Omega)/(p - q)}{\text{RSS}_\Omega/(n - p)}$$

Null hypothesis: the simplification to Ω implied by the simpler model, ω .

Under the null hypothesis, the F-statistic has an F-distribution with $p - q$ and $n - p$ degrees of freedom.

Leads to tests of the form: reject H_0 for $F > F_{p-q, n-p}^{(\alpha)}$.

Deriving this fact is beyond this class (take Linear Models), but let's try to appreciate its pieces.

Notes about the F-statistic

Example: Overall regression F-test

The overall regression F-test asks if any predictors are related to the response.

Full model: $Y = X\beta + \epsilon$, $\epsilon \sim N(0, \sigma^2 I)$

Reduced model: $Y = \beta_0 + \epsilon$

Null hypothesis: $H_0 : \beta_1 = \beta_2 = \dots = \beta_{p-1} = 0$

All the parameters (other than the intercept) are zero.

Alternative hypothesis: At least one parameter is non-zero.

Exercise: question #1 on handout

If there is **evidence against the null hypothesis**:

- ▶ The null is not true, or
- ▶ the null is true but we got unlucky, or
- ▶ the full model isn't true and the F-test is meaningless.

If there is **no evidence against the null hypothesis**:

- ▶ The null is true, or
- ▶ the null is false but we didn't gather enough evidence to reject it, or
- ▶ the full model isn't true and the F-test is meaningless.

Example: One predictor

Null hypothesis: $\beta_j = 0$

Equivalent to the t-test, reject null if

$$|t_j| = \left| \frac{\hat{\beta}_j}{\text{SE}(\hat{\beta}_j)} \right| > t_{n-p}^{\alpha/2}$$

In fact, in this case, $F = t_j^2$.

Exercise: questions #2 & #3 on handout

Other examples

- ▶ More than one parameter
- ▶ A subspace of the parameter space

Exercise: questions #4 & #5 on handout

We can't do F-tests when

- ▶ we want to test non-linear hypotheses, e.g. $H_0 : \beta_j \beta_k = 1$ (we might be able to make use of the Delta method, though)
- ▶ we want to compare non-nested models (find an example on the handout)
- ▶ the models fitted use different data