Some details to tidy up

ST552 Lecture 7

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Go over the estimation of σ

The trick to finding the expectation of e_i^2 is writing them as a linear combination of uncorrelated variables, ϵ_i.

Summary

For the linear regression model

$$Y = X\beta + \epsilon$$

where $\mathsf{E}\left(\epsilon\right) = 0_{n\times 1}$, $\mathsf{Var}\left(\epsilon\right) = \sigma^2 I_n$, and the matrix $X_{n\times p}$ is fixed with rank p.

The least squares estimates are

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

Furthermore, the least squares estimates are BLUE, and

$$\mathsf{E}\left(\hat{\beta}\right) = \beta, \qquad \mathsf{Var}\left(\hat{\beta}\right) = \sigma^2 (X^T X)^{-1}$$

$$\mathsf{E}\left(\hat{\sigma}^2\right) = \mathsf{E}\left(\frac{1}{n-p}\sum_{i=1}^{n} e_i^2\right) = \sigma^2$$

We have not used any Normality assumptions to show these properties.

Normality assumption

Assume $\epsilon \sim N(0, \sigma^2 I)$.

Important reminders:

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Leads to:

$$Y \sim N(\quad , \quad)$$

$$\hat{\beta} \sim N(\quad , \quad)$$

Inference on individual parameters

With the addition of the Normal assumption, it can be shown that

$$\frac{\hat{\beta}_j - \beta_j}{SE(\hat{\beta}_j)} \sim t_{n-p}$$

leads to the usual construction of tests and confidence intervals for single parameters.

Exercises