

Property of  $\hat{\beta}$ : The least squares estimates of  $\beta$  are unbiased

$$E(\hat{\beta}) = E((X^T X)^{-1} X^T Y)$$

$$= E\left((X^T X)^{-1} X^T \left( \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} \right)\right) \quad \text{plug in regression equation for } Y$$

$$= E\left( \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} + \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} \right) \quad \text{expand}$$

$$= E\left( \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} + \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} \right) \quad \begin{array}{l} \text{simplify} \\ \text{term on left} \\ A^{-1}A = I \end{array}$$

$$= \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} + E\left( \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} \right) \quad \text{linearity of expectation}$$

$$= \begin{array}{c} \phantom{Y} \\ \phantom{Y} \end{array} \quad \begin{array}{l} \text{regression assumptions} \\ E(\varepsilon) = 0 \end{array}$$