ST552 Final

Winter 2015

Answer the questions in the spaces provided on this exam.

Name: _____

- You have 110 minutes to complete the exam.
- There are 3 questions. Answer all of the questions.
- Please
  - do not look at the exam until I tell you and
  - stop writing when I announce that the exam is over.
- There is one page of statistical tables at the end of the exam. You may remove the page of tables if you desire.

Question	Points	Score
1	15	
2	15	
3	15	
Total:	45	

1. An experiment was conducted to explore the relationship between the *lifetime* (measured in days) and sexual activity of fruitflys.

   125 fruit flys were divided randomly into 5 treatment groups, each of 25 flys. Each treatment was designed to simulate a different level of sexual activity, with levels: *none*, *one*, *low*, *many* and *high*.

   The *thorax length* of each male was also measured (in mm) as this was known to affect lifetime.

   One observation in the *many* group was lost.

   The following model was fit:

   $$\log\left(\text{Lifetime}_i\right) = \beta_0 + \beta_1 \text{Thorax Length}_i + \beta_2 \text{one}_i + \beta_3 \text{low}_i + \beta_4 \text{many}_i + \beta_5 \text{high}_i + \epsilon_i$$

   where *one*, *low*, *many*, and *high* are indicator variables for the respective treatment groups, resulting in the following estimates and standard errors:

   $$\hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \\ \hat{\beta}_5 \end{pmatrix} = \begin{pmatrix} 1.84 \\ 2.72 \\ 0.05 \\ -0.12 \\ 0.09 \\ -0.42 \end{pmatrix}, \quad \text{SE}\left(\hat{\beta}\right) = \begin{pmatrix} 0.2 \\ 0.23 \\ 0.05 \\ 0.05 \\ 0.06 \\ 0.06 \end{pmatrix}, \quad \hat{\sigma} = 0.19$$

   (a) Construct a 95% confidence interval for the parameter, β_1. (2)

   Solution:
   95% CI: $\hat{\beta}_1 \pm t_{n-p}^{(0.975)} \text{SE}\left(\hat{\beta}_1\right)$
   $n - p = 124 - 6 = 118 \implies t_{n-p}^{(0.975)} = 1.97$

   $$\begin{aligned} \hat{\beta}_1 \pm t_{n-p}^{(0.975)} \text{SE}\left(\hat{\beta}_1\right) &= 2.72 \pm 1.97(0.23) \\ &= (2.2669, 3.1731) \end{aligned}$$

(b) Interpret the point estimate for β_1, in the context of the study on the **original scale** of lifetime. (`exp(2.72) = 15.18`) (3)

Solution: For flys under the same treatment, we estimate that an increase in thorax length of 1mm is associated with an increase in median lifetime of 1418%.
(OR) For flys under the same treatment that differ only by 1mm in thorax length, we estimate that the longer flys have a median lifetime of 15 times the median lifetime of the shorter flys.

(c) Are any additional assumptions (beyond the usual regression assumptions) required for your interpretation above? (2)

Solution: Yes, that on the log scale the response is symmetric about its mean.

(d) What additional information is required to construct a confidence interval on $\beta_2 - \beta_3$? (2)

Solution: $(X^TX)^{-1}$ or $\text{Cov}\left(\hat{\beta}_2, \hat{\beta}_3\right)$

(e) A more complicated model that treats *Thorax Length* as a categorical variable and includes interactions between the treatment groups and thorax length is also fit, with a resulting residual sum of squares (RSS) of 2.59 on 77 degrees of freedom.

i. What is the value of the F-statistic from an Extra Sum of Squares F-test comparing this model to the one above? (4)

Solution:

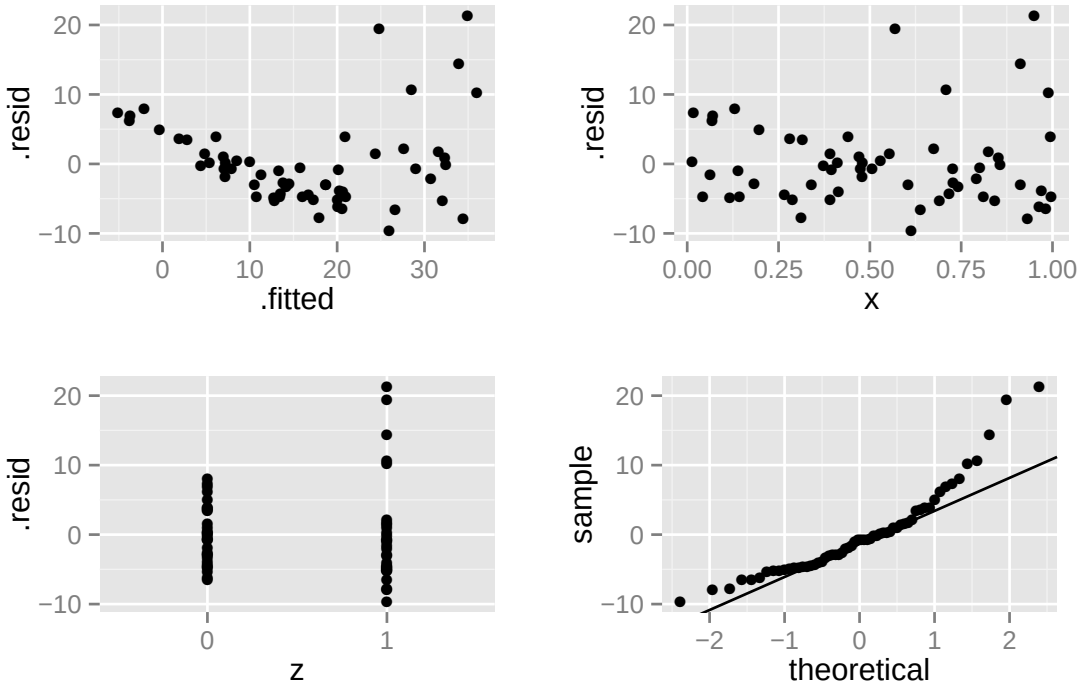
$$\begin{aligned}
\text{RSS}_R &= \hat{\sigma}^2 * \text{d.f.}_R = 0.19^2(118) = 4.26 \\
F &= \frac{(\text{RSS}_R - \text{RSS}_F)/(\text{d.f.}_R - \text{d.f.}_F)}{\text{RSS}_F/\text{d.f.}_F} \\
&= \frac{(4.26 - 2.59)/(118 - 77)}{2.59/77} \\
&= \frac{0.0407}{0.0336} \\
&= 1.21
\end{aligned}$$

ii. What is the special name for this F-test, and what would we conclude? **Errata:** Assume p-value = 0.15. (2)

Solution: This is a lack-of-fit F-test. We conclude there is no evidence of a lack of fit.

2. (a) The following residual plots come from a regression of the form:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \epsilon_i \quad i = 1, \ldots, 60$$



i. Name the assumption that appears to be violated. (2)

Solution: Linearity or constant spread. (pick one)

ii. Describe the evidence you see in the plots for the violation. (1)

Solution: Linearity: In the residuals versus fitted values plot, the residuals are systemically above the line between 0 and 5, below the line 5 to 20, then above > 20.
Constant spread: In the residuals versus fitted values plot, the residuals have gradually increasing spread moving from left to right - funnel/fan shaped.

iii. What are the consequences of proceeding with inference ignoring the violation? (1)

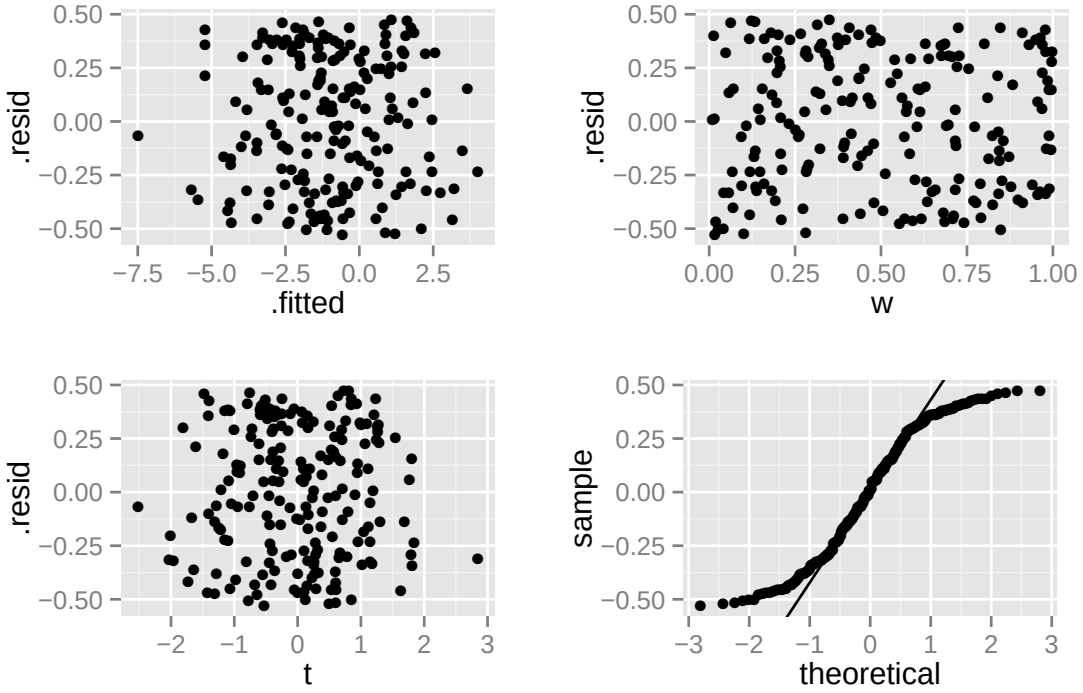
Solution: Linearity: Biased or misleading estimates.
Constant spread: Unbiased estimates but prediction variances not appropriate.

iv. How would you suggest proceeding? (1)

Solution: Try (log) transforming the response, might solve both problems.

(b) The following residual plots come from a regression of the form:

$$y_i = \beta_0 + \beta_1 w_i + \beta_2 t_i + \epsilon_i \quad i = 1, \ldots, 200$$



i. Name the assumption that appears to be violated. (2)

Solution: Normality of errors.

ii. Describe the evidence you see in the plots for the violation. (1)

Solution: Residuals in Q-Q plot don't lie near the line.

iii. What are the consequences of proceeding with inference ignoring the violation? (1)

Solution: Prediction intervals are innappropriate. Can still rely on Normal sampling distribution of estimates with large samples.

iv. How would you suggest proceeding? (1)

Solution: Unless prediction intervals are important, proceed with inference since the sample size is moderately large (200), we can rely on the CLT.

(c) A client has run diagnostics on a regression analysis and identified a single observation with very high leverage, but she admits she doesn't know what leverage is or how to proceed.

i. What does *high leverage* mean? (1)

> **Solution:** That the observation has an unusual combination of explanatory variable values compared to the other observations.

ii. Sketch a scatterplot that includes a point that has **high leverage** but is **not influential**. (Make sure you label your axes, clearly identify the point of interest, and label any fitted lines you add) (1)

iii. Sketch a scatterplot that includes a point that has **high leverage** and **is influential**.(Make sure you label your axes, clearly identify the point of interest, and label any fitted lines you add) (1)

iv. How would you advise your client to proceed? (2)

Solution: (Anything sensible) Is the observation also influential? If not, can probably proceed since conclusions won't be sensitive to whether this observation is included or excluded.
If it is influential, study it further. Is there reason to exclude it?
Could exclude and make restricted (based on reduced range of explantory values) inferences.

3. (a) i. Describe what is meant by **multicollinearity**. (2)

Solution: Multicollinearity refers to the situation when one or more explanatory variables are close to being a linear combination of the others.

ii. What are the consequences of multicollinearity? (2)

Solution: Exact collinearity, $X^T X$ is not invertable.
Multicollinearity results in large standard errors on individual parameters reflecting the uncertainty in attributing variation in the response to variables that are correlated.

(b) i. In one sentence, describe what is meant by **variable selection** in the context of multiple linear regression. (1)

Solution: Variable selection refers to the process of selecting a subset of variables for inclusion in the regression model from some larger set.

ii. Give a reason why variable selection might be recommended. (2)

Solution: Parsimony is desired.
Prediction is important and there is reason to beleive the large set includes some variables that aren't related to the response.

iii. Give a reason why variable selection might be avoided. (2)

Solution: The goal is to make inferential statements about a particular variable.

(c) Some extensions to multiple linear regression include: (6)

- Robust regression
- Generalized least squares
- Regularized regression
- Logistic regression
- Non-linear regression

Pick **two** methods from the above list and describe how they differ from the usual case of linear regression.